Coupling of Current and Flow Relaxation in Reversed-Field Pinches Due to Two-Fluid Effects

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2012 Fusion Energy Conference

October 8-12, 2012   San Diego, CA

Objective

To understand effects that couple current-profile and flow-profile relaxation (momentum transport) during relaxation events in the Madison Symmetric Torus (MST).

Outline

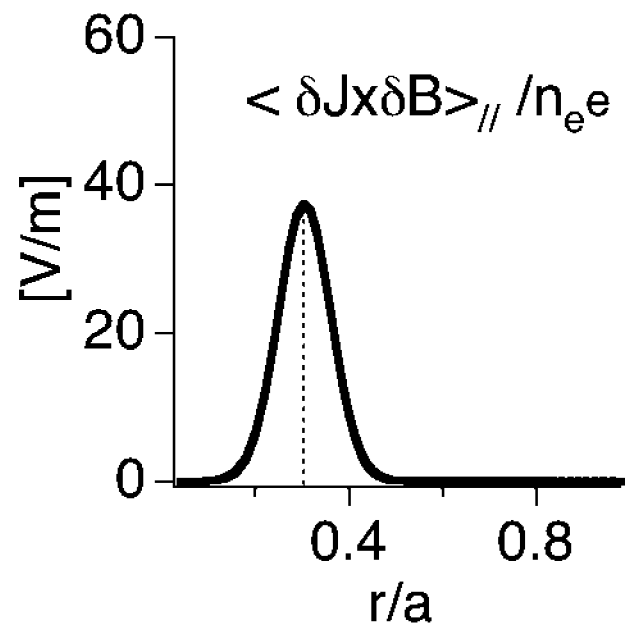
- Introduction
- Modeling
- Linear results
- Nonlinear island evolution
- Dynamo and momentum transport in multi-helicity relaxation
- Multi-helicity relaxation with background flow
- Discussion and conclusions

Introduction: Measurements on MST point to the importance of two-fluid effects during relaxation events.

- In standard MST operation, magnetic relaxation occurs during discrete sawtooth events.

- Laser polarimetry measurements of δj_ϕ for the (1,6) mode, and fitted $\delta\mathbf{b}$ profiles show a correlation that implies significant Hall dynamo. [Ding, *et al.*, PoP **13**, 112306].

- MST parameters have ion-sound-gyroradius ($\rho_s = c_s/\Omega_i$) comparable to the resistive skin depth, so two-fluid linear tearing effects are expected [Mirnov, *et al.*, PoP **11**, 4468, for example].



Hall dynamo from (1,6) mode, inferred with multi-chord laser polarimetry measurements on MST.

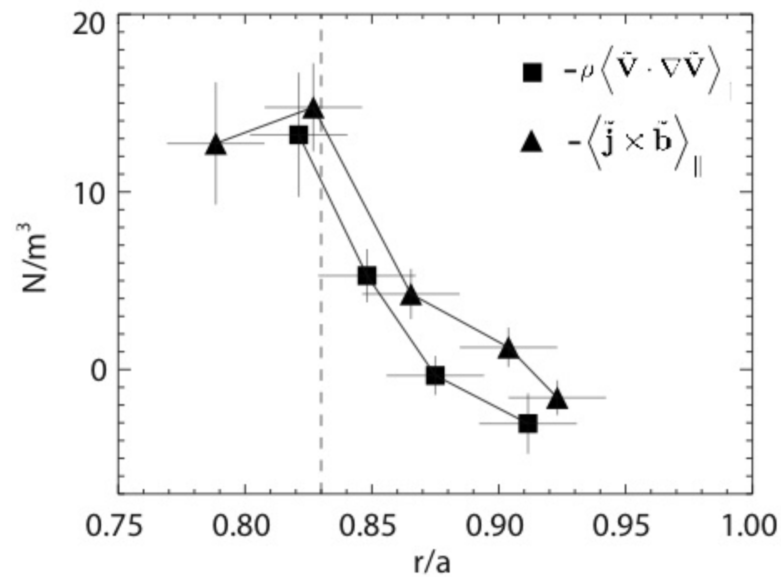
Probe measurements from MST's edge plasma also indicate the significance of two-fluid effects.

• The existence of Hall dynamo and net parallel Lorentz force density from fluctuations are essentially equivalent.

• Kuritsyn, *et al*., used an array of magnetic coil triplets to measure Maxwell-stress profiles in the edge of MST [PoP **16**, 55903].

• The group also found Reynolds stress contributions from flow fluctuations, measured by Mach probes.

• With respect to momentum transport, the effects largely cancel.



Fluctuation-induced Lorentz and inertial force densities from probe measurements during a relaxation event.

Modeling: We apply two-fluid modeling to investigate these macroscopic RFP dynamics.

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{V}) = 0$$ particle continuity

$$mn\left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla\right)\mathbf{V} = \mathbf{J}\times\mathbf{B} - \nabla\sum_{\alpha} nT_{\alpha} - \underline{\nabla\cdot\underline{\Pi}}$$ flow evolution

$$\frac{3}{2}n\left(\frac{\partial}{\partial t} + \mathbf{V}_{\alpha}\cdot\nabla\right)T_{\alpha} = -nT_{\alpha}\nabla\cdot\mathbf{V}_{\alpha} - \nabla\cdot\mathbf{q}_{\alpha} + Q_{\alpha}$$ temperature evolution

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla\times\left[\eta\mathbf{J} - \mathbf{V}\times\mathbf{B} + \underline{\frac{1}{ne}\mathbf{J}\times\mathbf{B} - \frac{T_e}{ne}\nabla n + \frac{m_e}{ne^2}\frac{\partial}{\partial t}\mathbf{J}}\right]$$ Faraday's / Ohm's law

$$\mu_0\mathbf{J} = \nabla\times\mathbf{B}$$ low-ω Ampere's law

$$\nabla\cdot\mathbf{B} = 0$$ divergence constraint

- Initial-value computations with the two-fluid system are solved using the NIMROD code [JCP **229**, 5803 (2010)].

The closure for stress ($\underline{\Pi}$) is a combination of Braginskii ion gyroviscosity and isotropic viscous stress.

$$\underline{\Pi}_{\mathrm{gv}} = \frac{m_{\mathrm{i}} p_{\mathrm{i}}}{4eB}\left[\hat{\mathbf{b}} \times \underline{\mathbf{W}} \cdot \left(\underline{\mathbf{I}} + 3\hat{\mathbf{b}}\hat{\mathbf{b}}\right) - \left(\underline{\mathbf{I}} + 3\hat{\mathbf{b}}\hat{\mathbf{b}}\right) \cdot \underline{\mathbf{W}} \times \hat{\mathbf{b}}\right], \qquad \left(\underline{\mathbf{W}} \equiv \nabla\mathbf{V} + \nabla\mathbf{V}^{\mathrm{T}} - \frac{2}{3}\underline{\mathbf{I}}\nabla \cdot \mathbf{V}\right)$$

$$\underline{\Pi}_{\perp} \sim -\frac{3 p_i m_i^2}{10 e^2 B^2 \tau_i}\underline{\mathbf{W}} \quad , \quad \text{treated as } -n m_i \nu_{iso}\underline{\mathbf{W}}$$

The computations presented here have finite but uniform background pressure, representing conditions in the RFP core without detailed transport modeling.

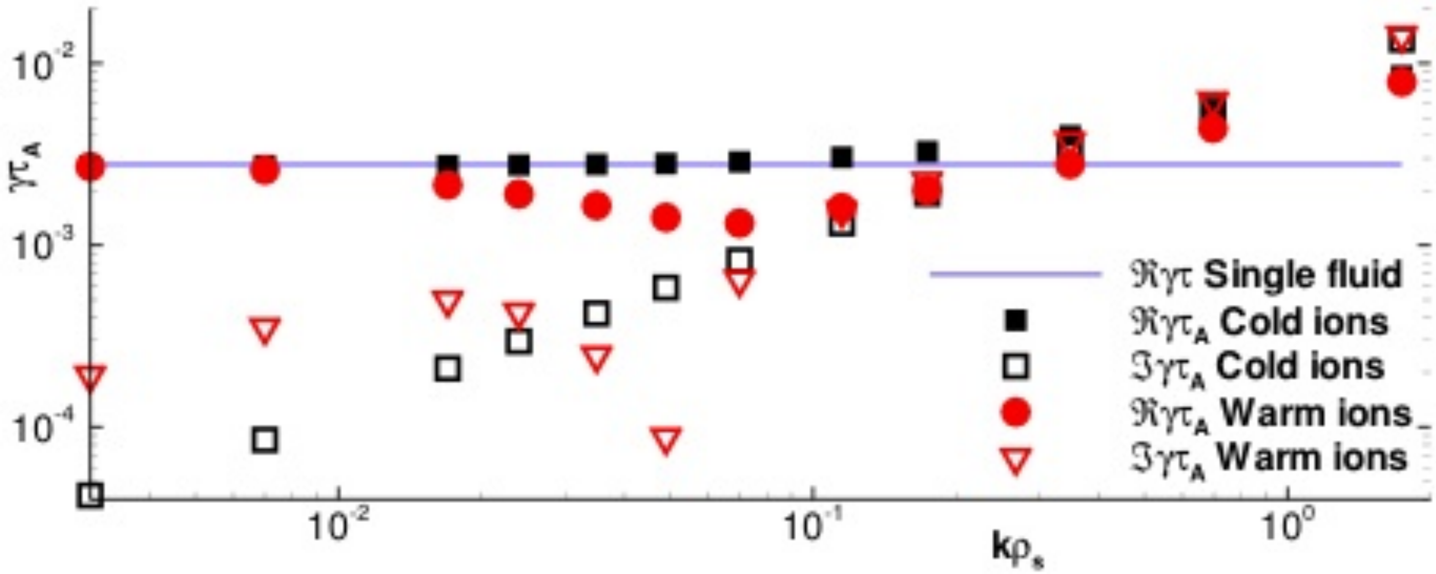
$$\mathbf{q} = -n\chi_{iso}\nabla T$$

Linear results: 'Fast tearing' was expected, a slower intermediate regime was not.

- Analytical asymptotic results for linear tearing in a slab with uniform equilibrium pressure finds $\gamma > \gamma_{MHD}$ for $\rho_s > \delta$. [Mirnov; also Ahedo and Ramos, PPCF **51**, 55018.]
- Our linear slab computations [Sovinec and King, JCP **229**, 5803 (2010)] reproduce the analytics.
- Cold-ion results for paramagnetic pinch equilibria also show growth rates that are larger than resistive-MHD.

	MST	comput.
β	0.03	0.1
kd_i	0.7	0.01-6.0
$k\rho_s$	0.05	0.003-1.7

Warm-ion (gyroviscous, $T_i = T_e$) results for cylindrical geometry find $\gamma < \gamma_{MHD}$ at intermediate ρ_s-values.



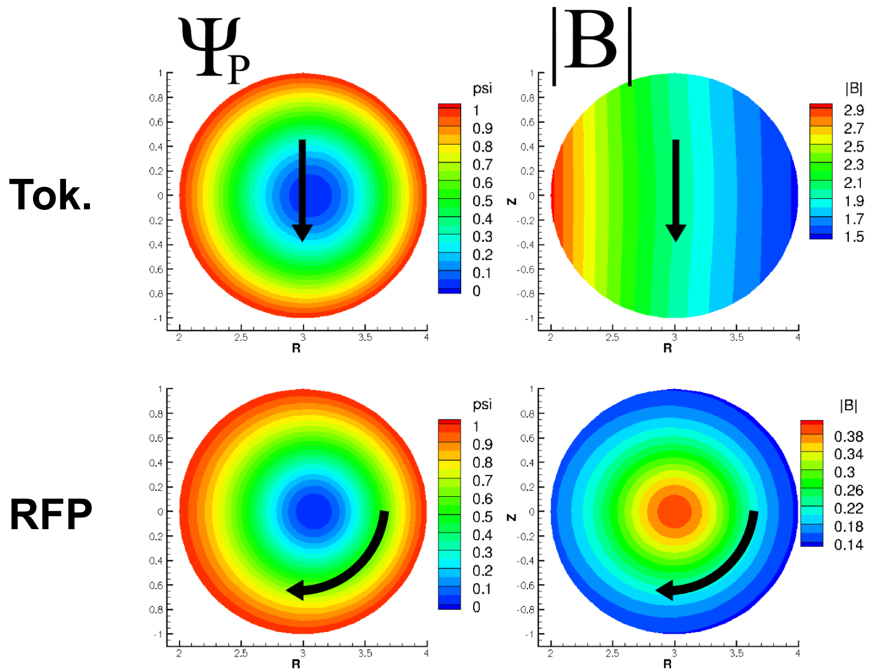
Growth rates (solid) and frequencies (open) from warm-ion (red) and cold-ion (black) computations with NIMROD. [log-log scale]

- The effect does not occur for warm-ion computations in slab geometry.

Our analysis of the ion gyroviscous stress indicates drift-tearing effects from ∇B and poloidal curvature.

• Expanding $\hat{\mathbf{b}}_0$ and **k** about the resonance in tearing ordering and analyzing $2\hat{\mathbf{b}}_0 \times \boldsymbol{\kappa}_0 \cdot \nabla \cdot \tilde{\Pi}$ and $-\hat{\mathbf{b}}_0 \cdot \nabla \times \nabla \cdot \tilde{\Pi}$ contributions to parallel vorticity evolution finds a drift contribution with frequency [King, *et al.*, PoP **18**, 42303]:

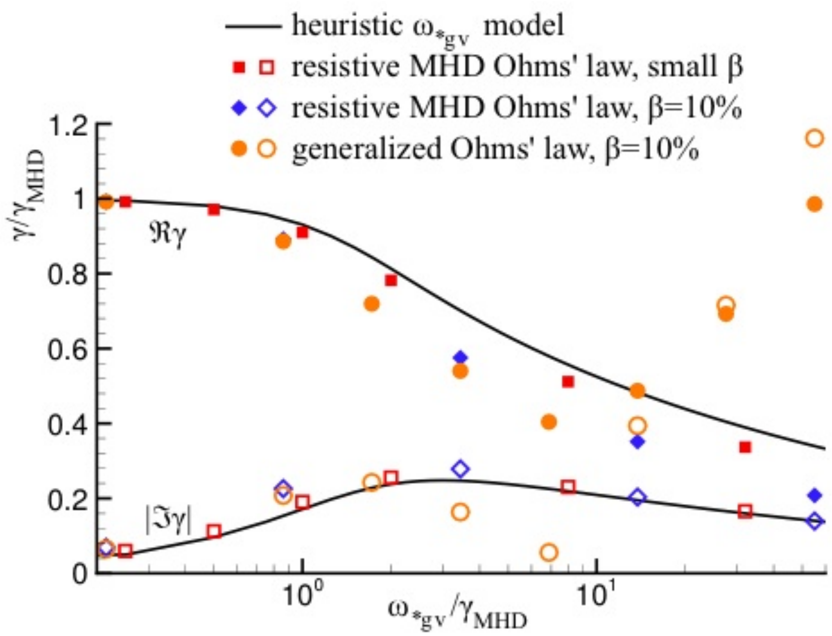
$$\omega_{*gv} = \frac{k_\perp}{m_i n_0} \frac{p_{i0}}{\Omega_{i0}} \left(\frac{3}{2r} \frac{B_{\theta 0}^2}{B_0^2} - \frac{B_0'}{B_0} \right)$$



Poloidal flux (left) and *B* (right) for tokamaks (top) and RFPs (bottom). Arrows show projections of ∇B and curvature drifts.

A simplified model with gyroviscous stress and resistive-MHD Ohm's law reproduces the stabilizing effect.

• The derivation also neglects compressive responses from pressure, and the dispersion relation is just $\gamma^4\left(\gamma - i\omega_{*gv}\right) = \gamma_{MHD}^5$.



Two-fluid computations and heuristic dispersion relation (drift & resistive-MHD Ohm's law) agree until KAW reconnection is significant, $\rho_s \sim \delta$.

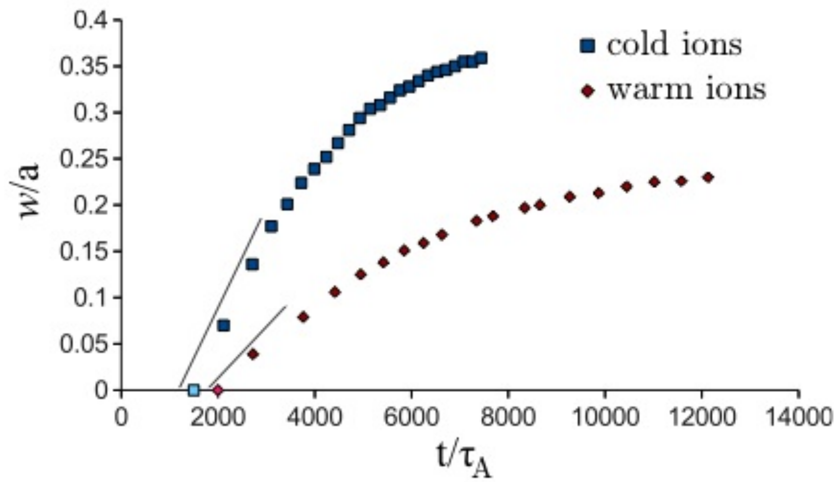
• Warm ions in pinch profiles lead to drift-tearing, even with ∇p_0=0.

Nonlinear single-helicity: Nonlinear island saturation with cold ions matches MHD.

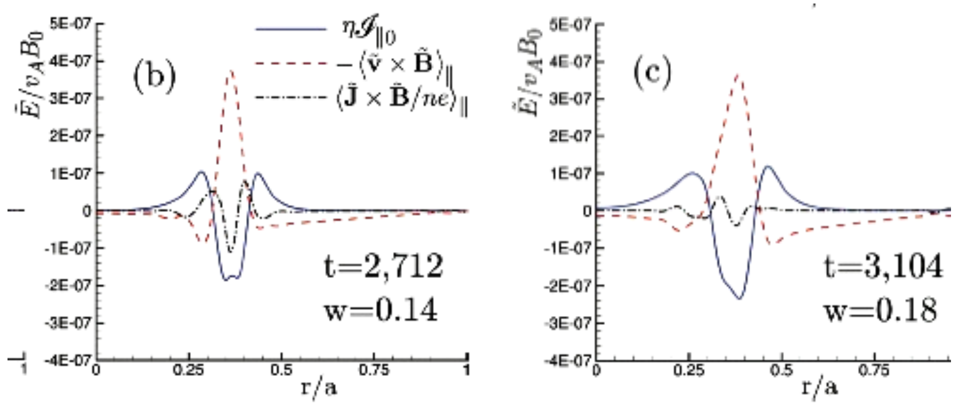
- The computations have R/a = 0.51 to prohibit multi-helicity dynamics.

- Coupling to $m \neq 1$ components is allowed but is not significant in the results.

- We observe that island evolution reverts to MHD dynamics, i.e. vanishing Hall effect, when the island width exceeds the ion skin depth.



Island-width evolution shows a Rutherford stage for cold and warm ions.



Hall dynamo effect in cold-ion evolution vanishes when *w* exceeds d_i (0.17a).

With warm ions, saturation occurs at smaller width.

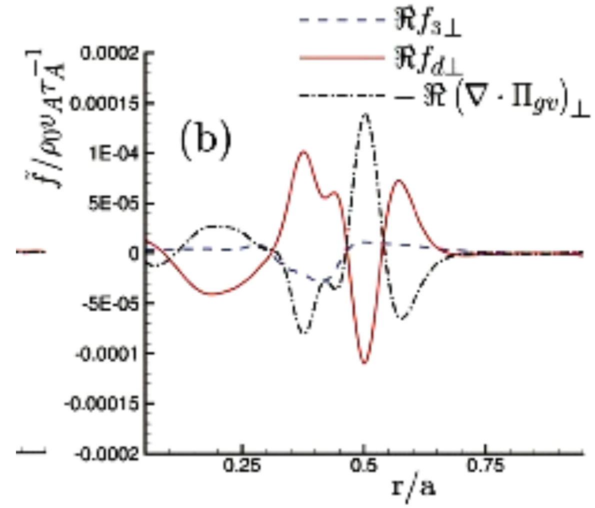
• Forces from gyroviscous stress supplement Rutherford's 3rd-order Lorentz force to balance the 1st-order drive.

• The important gv forces result from flows that are out of phase with the standard reconnection flow pattern.

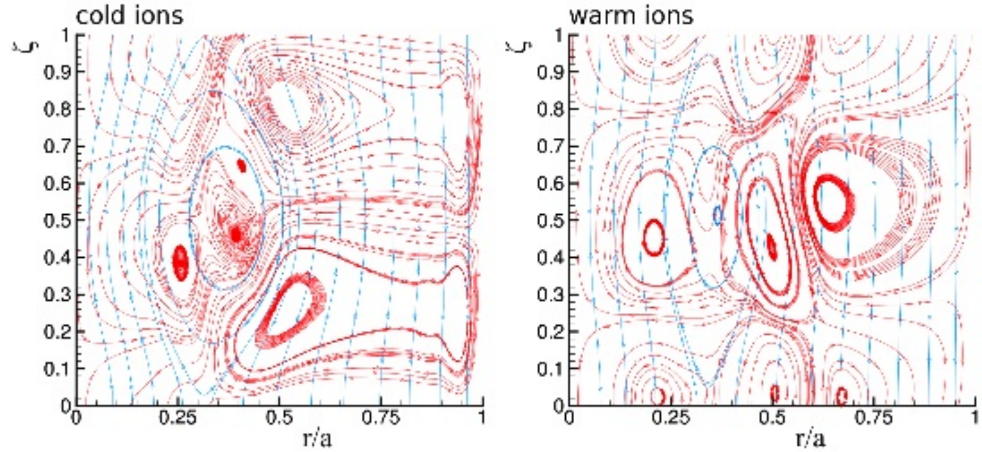
• The out-of-phase flows are influenced by Lorentz forces from perturbed currents resulting from electron-ion decoupling (Hall effect).

• The flows are insensitive to S and viscous dissipation.

• Unlike tokamak diamagnetic drift-tearing, transport does not eliminate large-*w* FLR [King, PoP **18**, 42303].



Comparison of 3rd-order, 1st-order, and gyroviscous forces at saturation.



Saturated helical flow streamtraces show different phases for cold-ion (left) and warm-ion (right) results.

Nonlinear multi-helicity: Results at realistic R/a are more directly relevant to standard operation.

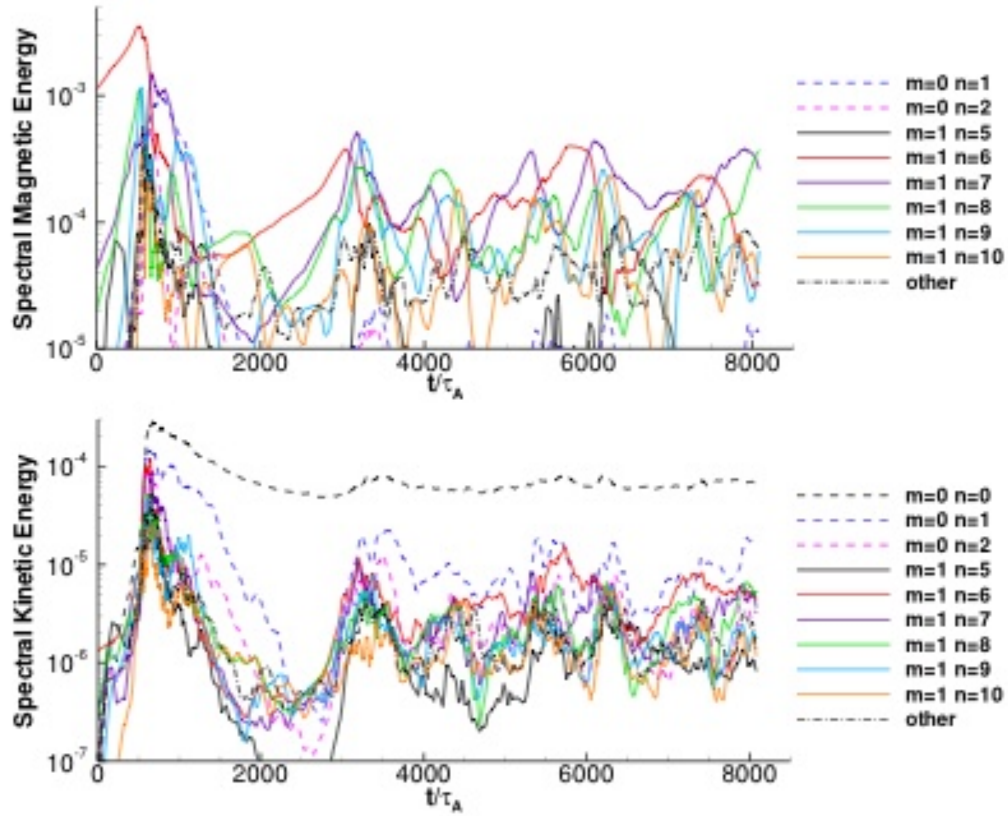
- Our multi-helicity two-fluid computations have R/a=3 in cylindrical geometry, Hall effect, and gyroviscous stress from warm ions.

- Plasma parameters have S=8×10^4, Pm=1, β=0.1, and ρ_s=0.05a.

- Overall evolution at Θ=1.6 shows familiar fluctuation dynamics and field reversal

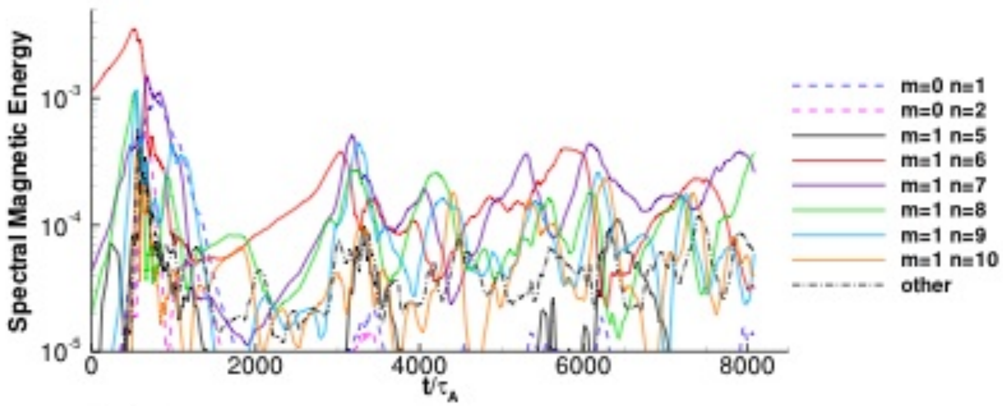
- Dynamics remain nearly 'force-free.'

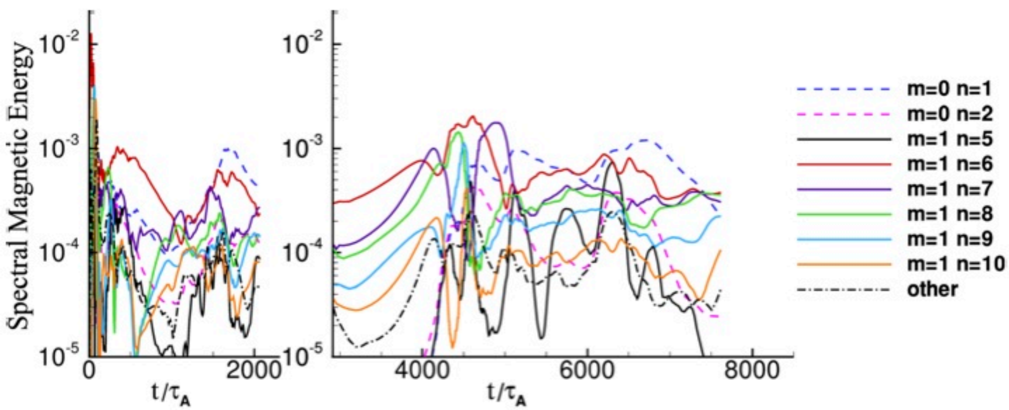
- Magnetic energy in *m*=0 fluctuations is relatively weak after the first event.



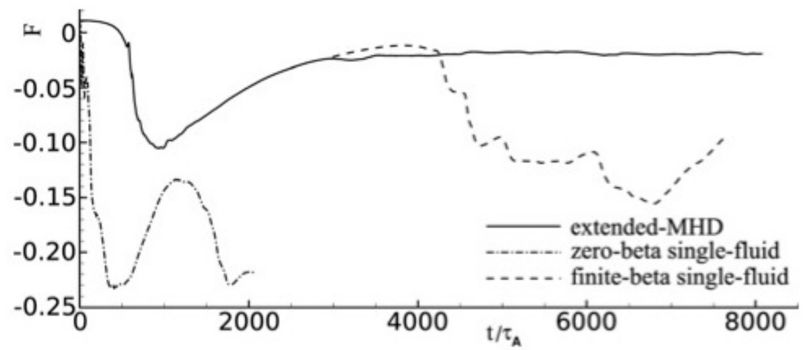
Evolution of fluctuation energies in the two-fluid computation.

MHD computations of the same conditions have larger core fluctuation amplitudes and more nonlinear coupling.



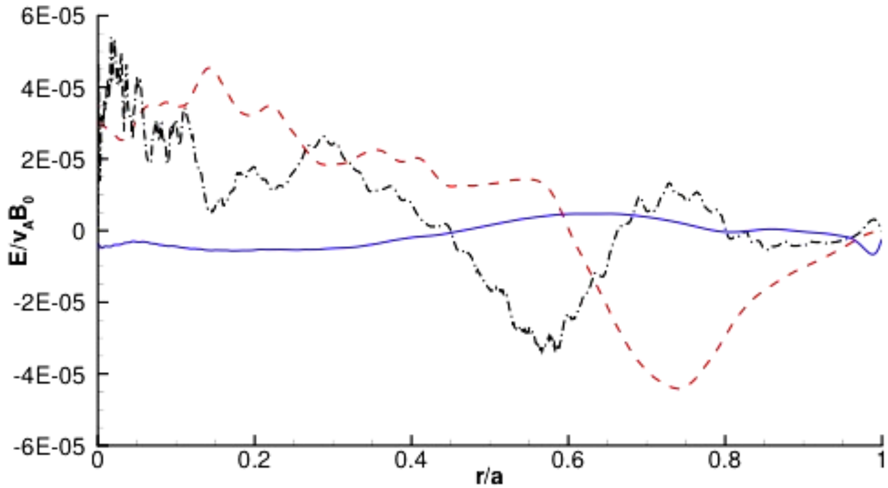


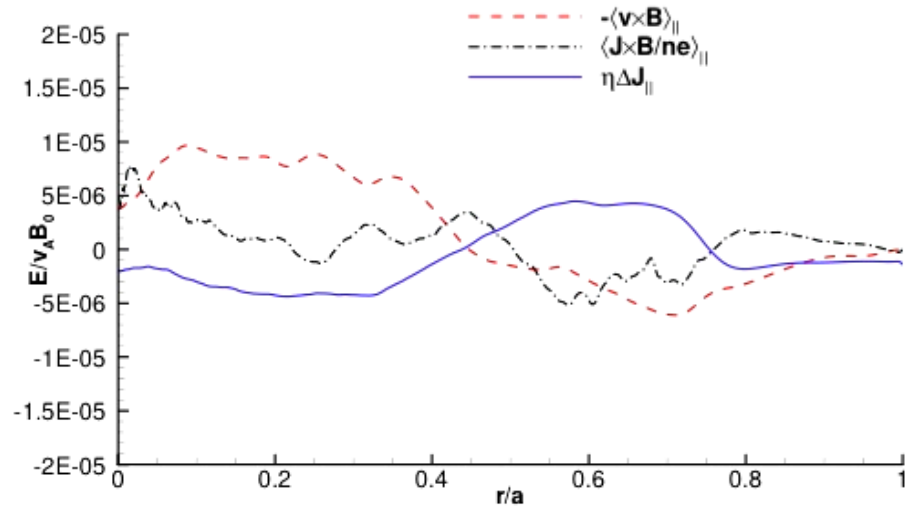
MHD computations started at *t*=0 with 0-β (below left) and at later time with finite-β (below right) develop larger *m*=0 fluctuations than the two-fluid model (top).



Smaller fluctuation levels in the two-fluid computation leads to less reversal than in the MHD computations for the same conditions.

Two-fluid multi-helicity computations with (ρ_s/a)-values relevant to MST exhibit Hall dynamo during relaxation.

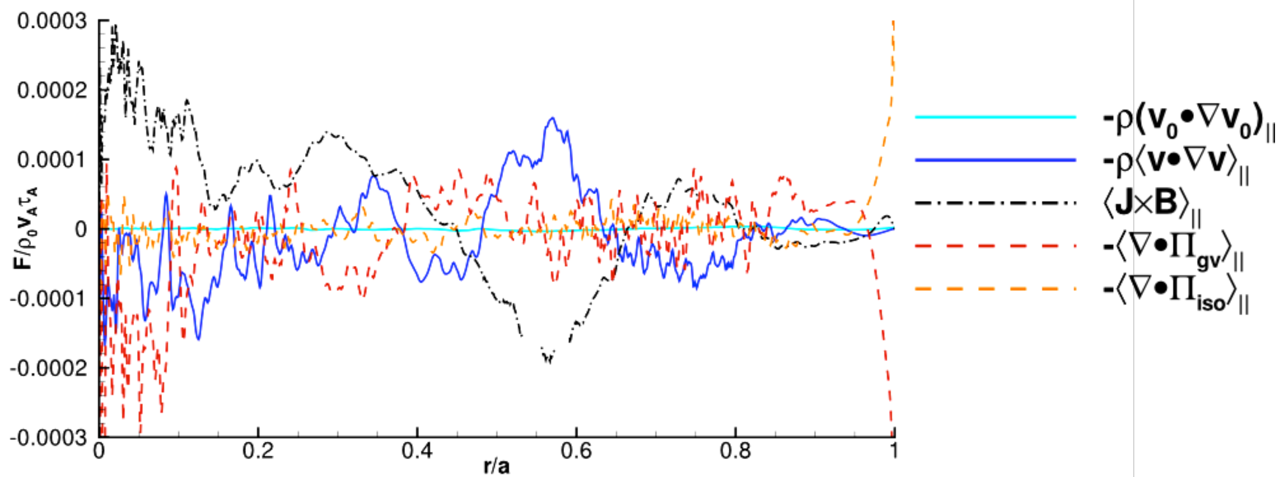




Hall dynamo is evident and comparable in magnitude to the MHD dynamo during the initial (left) and a subsequent (right) relaxation event.

- Our computations at ion sound gyroradius (c_s/Ω_i) ρ_s=0.05a show significant Hall and MHD dynamo effects that are comparable in magnitude during a relaxation event.
- As in single-fluid computation, the combined dynamo emfs act to reduce parallel current in the core and drive it near the edge.
- Evaluating for MST, $5\times10^{-5}v_AB_0$ is 40 V/m, so the predicted magnitude is comparable to the Hall emf measured by Ding et al. [PRL **93**, 2004].

There is also significant momentum transport during two-fluid relaxation with m=0 mode activity.

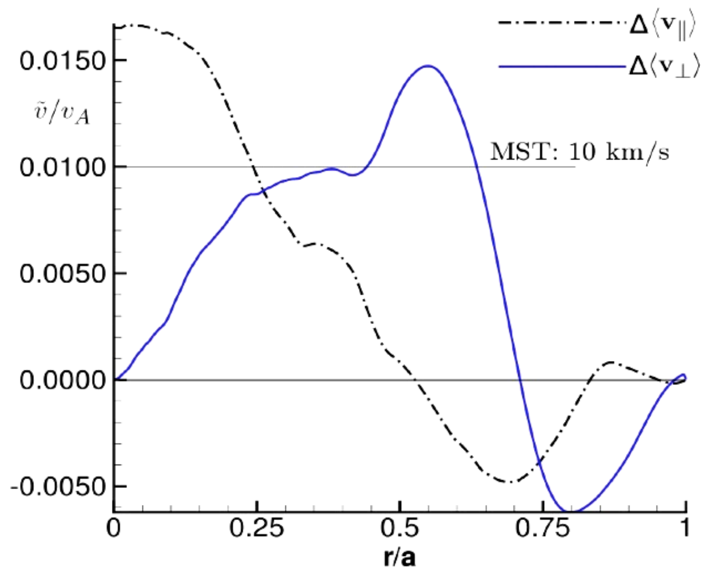


Force densities that affect simulated net parallel flow during a relaxation event, including contributions from Maxwell stress (black) and Reynolds stress (blue).

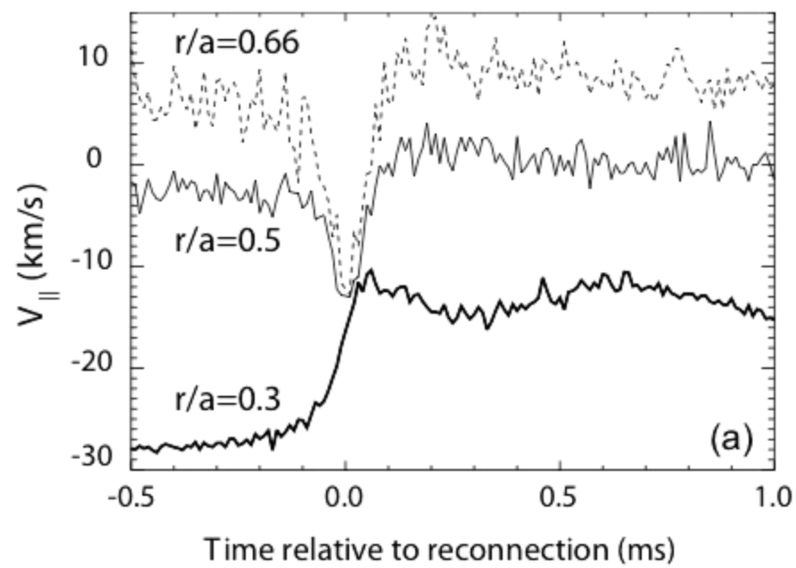
- Simulations show transport of parallel momentum driven by fluctuation-induced forces from Maxwell, Reynolds, and gyroviscous stresses.
- The fluctuation-induced Maxwell stress is linked to the Hall dynamo.
- Similar to measurements by Kuritsyn et al. [Phys. Plasmas **16**, 2009] in the edge of MST, the Maxwell and Reynolds stresses tend to cancel.
- The warm-ion computations also show significant gyroviscous forces.

Similar to MST, the large relaxation event increases parallel flow in the core and reduces it near the edge.

- The net change in flow profile is only significant with two-fluid effects.
- These simulations do not include other transport effects that maintain flow profiles.
- The relevance of analytical two-fluid relaxation theory [papers by Steinhauer, Hegna, for example] is intriguing but needs further study.



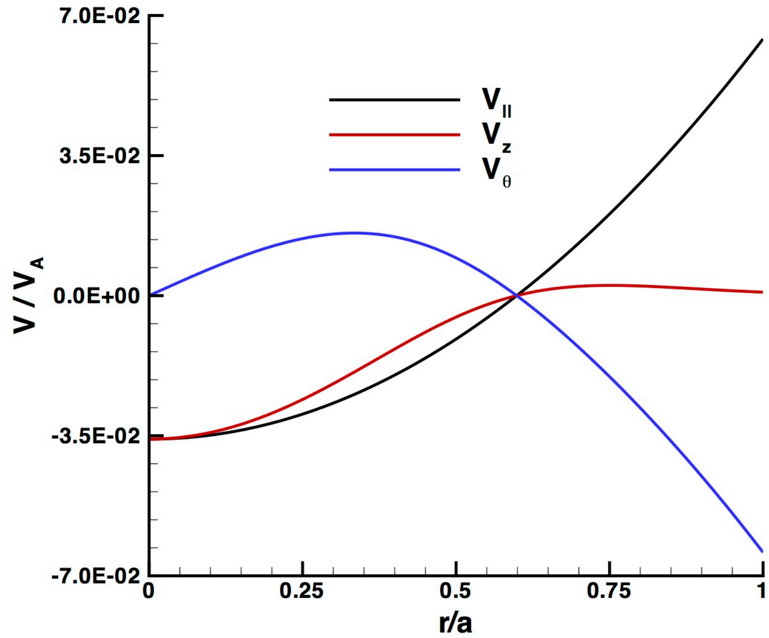
Profiles of parallel (black) and perpendicular (blue) flow generated by the first simulated relaxation event.



Temporal evolution of parallel flow at three radii in MST. [Kuritsyn, PoP 16, 55903]

Multi-helicity with background flow: MST develops a parallel-flow profile between events.

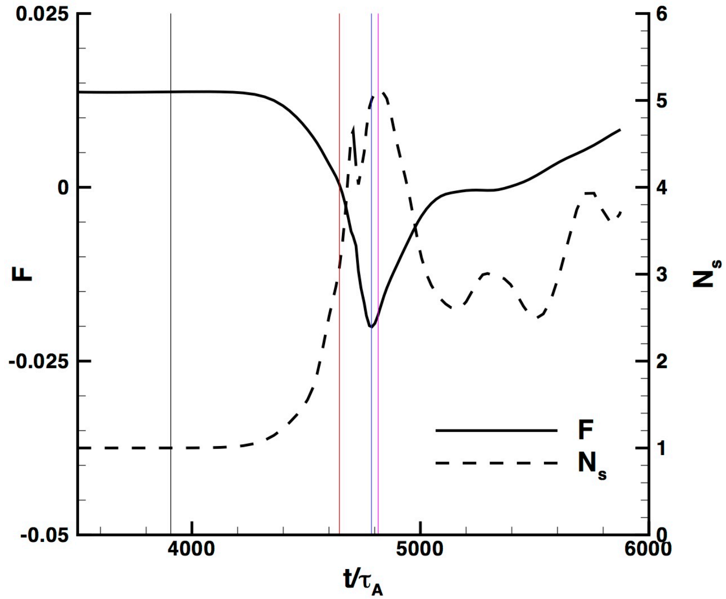
- We use a background parallel-flow profile that matches the experimental measurements between events at the probe locations shown above.
- Computing with S=5000 facilitates scanning different configurations.



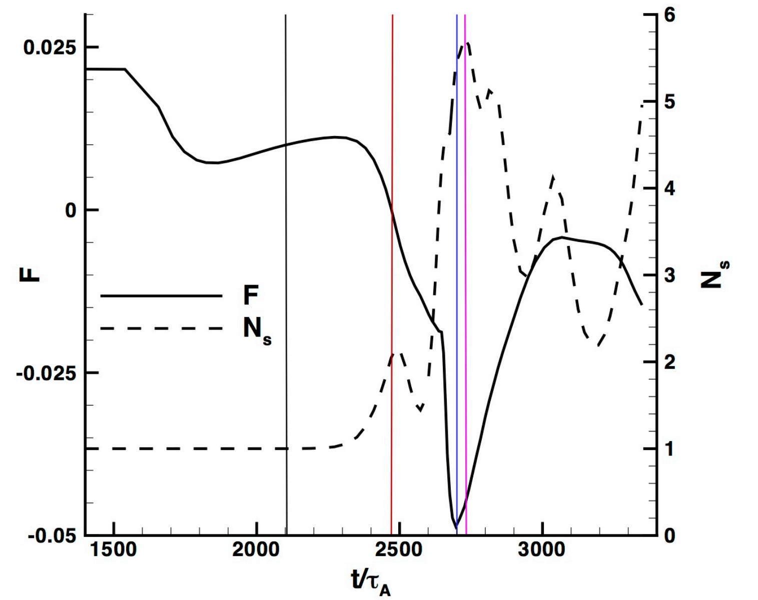
This background parallel-flow profile is used in all of the following results.

Background flow breaks the symmetry of relaxation relative to positive and negative current.

- Results with negative current, $\mathbf{J}_0 \cdot \mathbf{B}_0 < 0$, exhibit approximately twice as much field-reversal during the first relaxation event.
- Somewhat more nonlinear coupling occurs, as evident from the larger spectral width, $N_s \equiv \left(\sum_n W_{1,n}\right)^2 \Big/ \sum_n W_{1,n}^2$, where $W_{1,n}$ is the magnetic fluctuation energy in the (m=1,n) component.



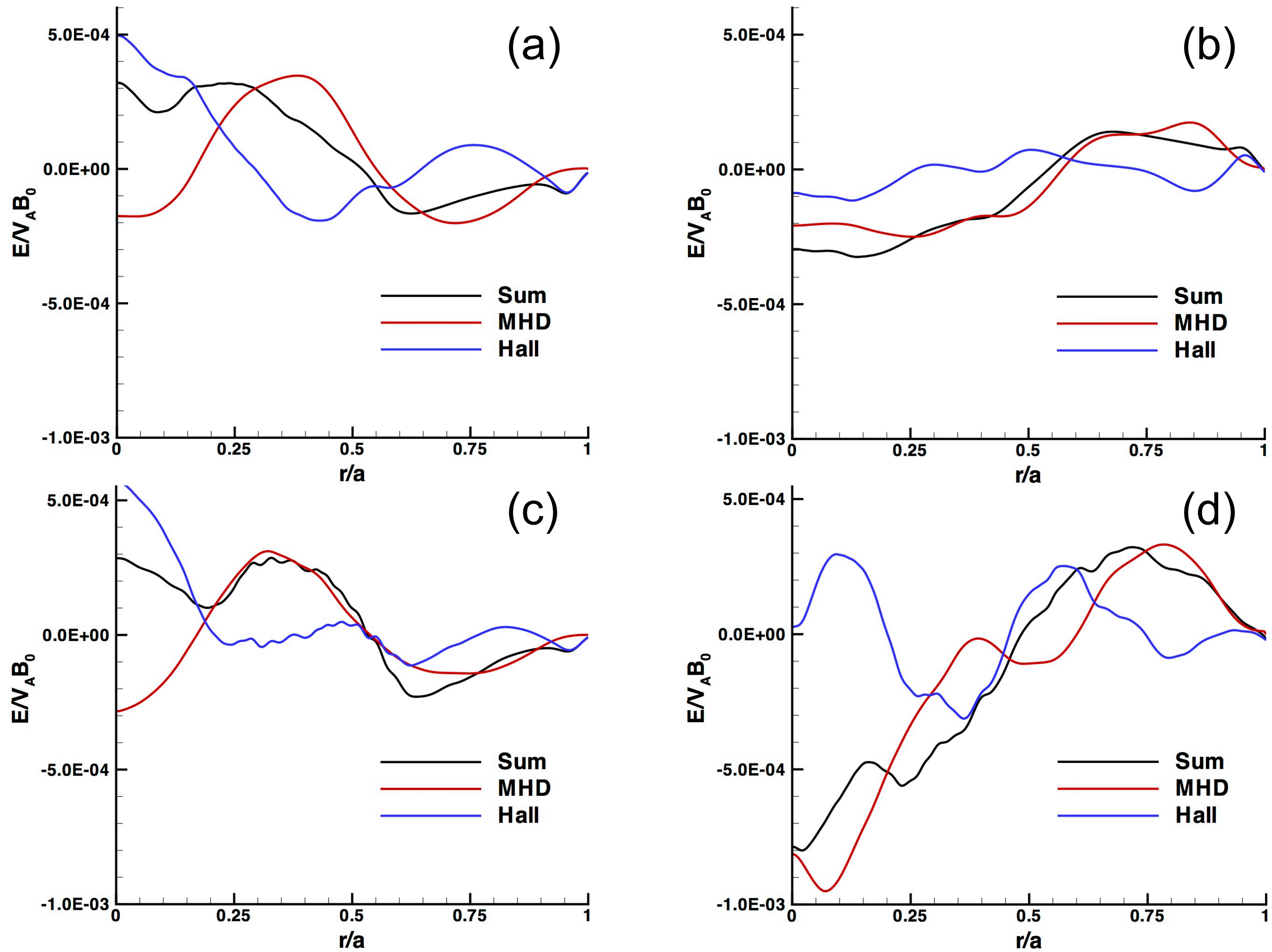
Evolution of reversal parameter (*F*) and spectral width (*N*$_s$) for positive current.



Evolution of *F* and *N*$_s$ with negative current.

Background flow alters Hall and MHD dynamo effects.

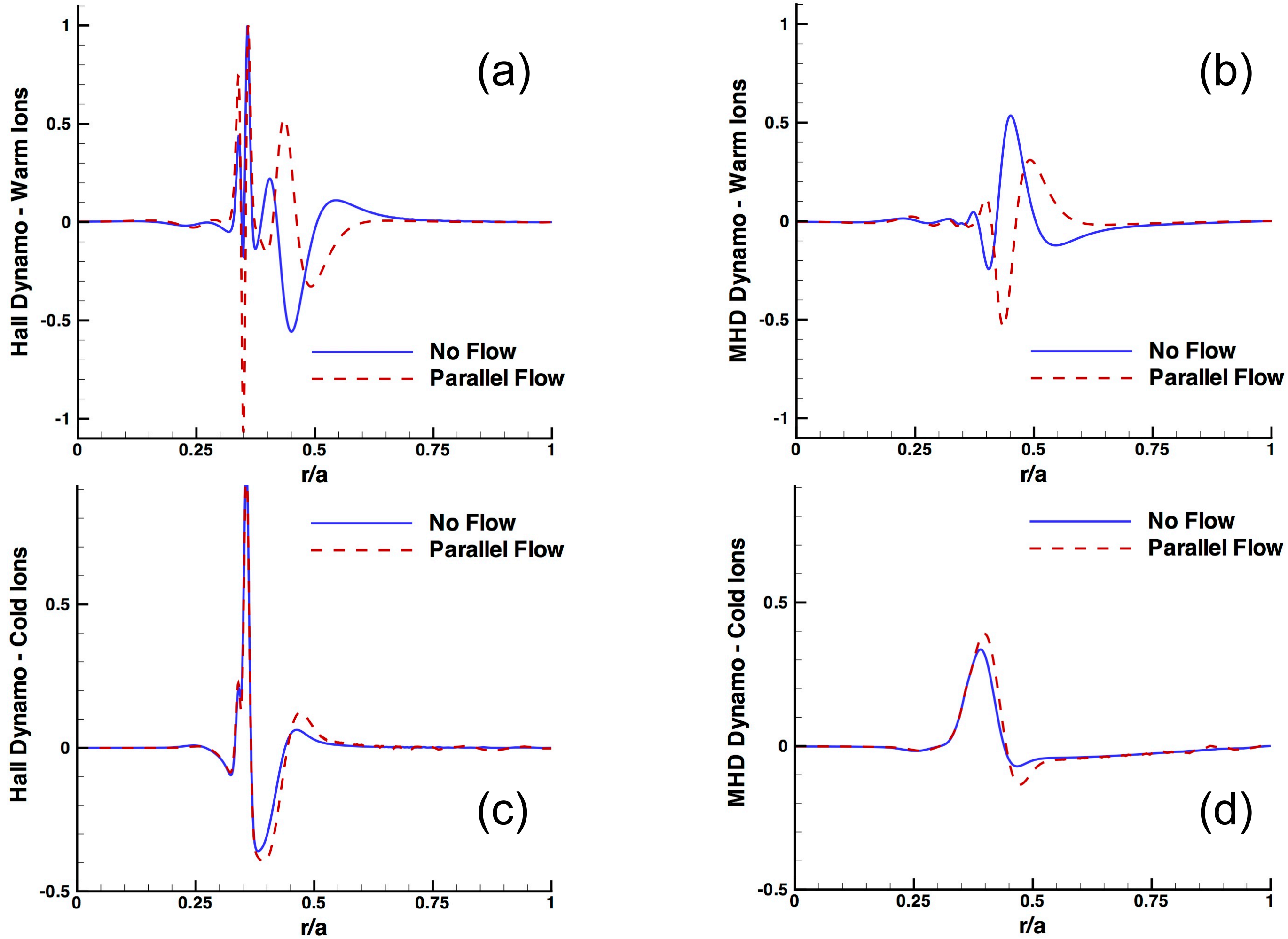
- At reversal the sum is similar for the two current orientations, but the contributions oppose each other with positive current.



Dynamo contributions at initial reversal with (a) $J_0 \bullet B_0>0$ and (b) $J_0 \bullet B_0<0$ and at greatest reversal with (c) $J_0 \bullet B_0>0$ and (d) $J_0 \bullet B_0<0$.

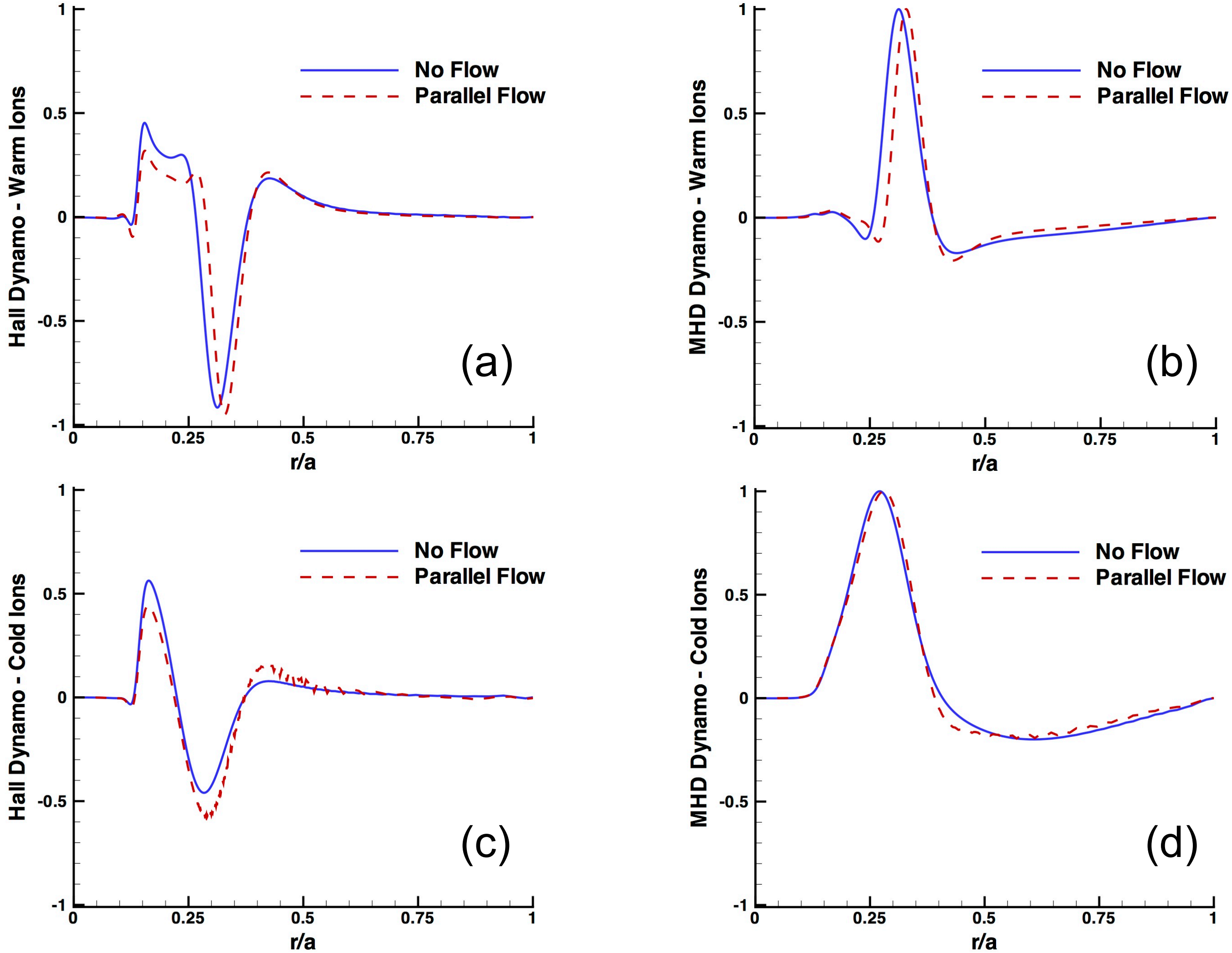
Background flow also affects quasilinear dynamo.

- This set of quasilinear profiles is computed from linear tearing-mode (Δ' =48) eigenfunctions for S=80,000, Pm=0.1.



Dynamo effects from tearing-mode results with $J_0 \bullet B_0 > 0$ (a) Hall with warm ions, (b) MHD with warm ions, (c) Hall with cold ions, (d) MHD with cold ions.

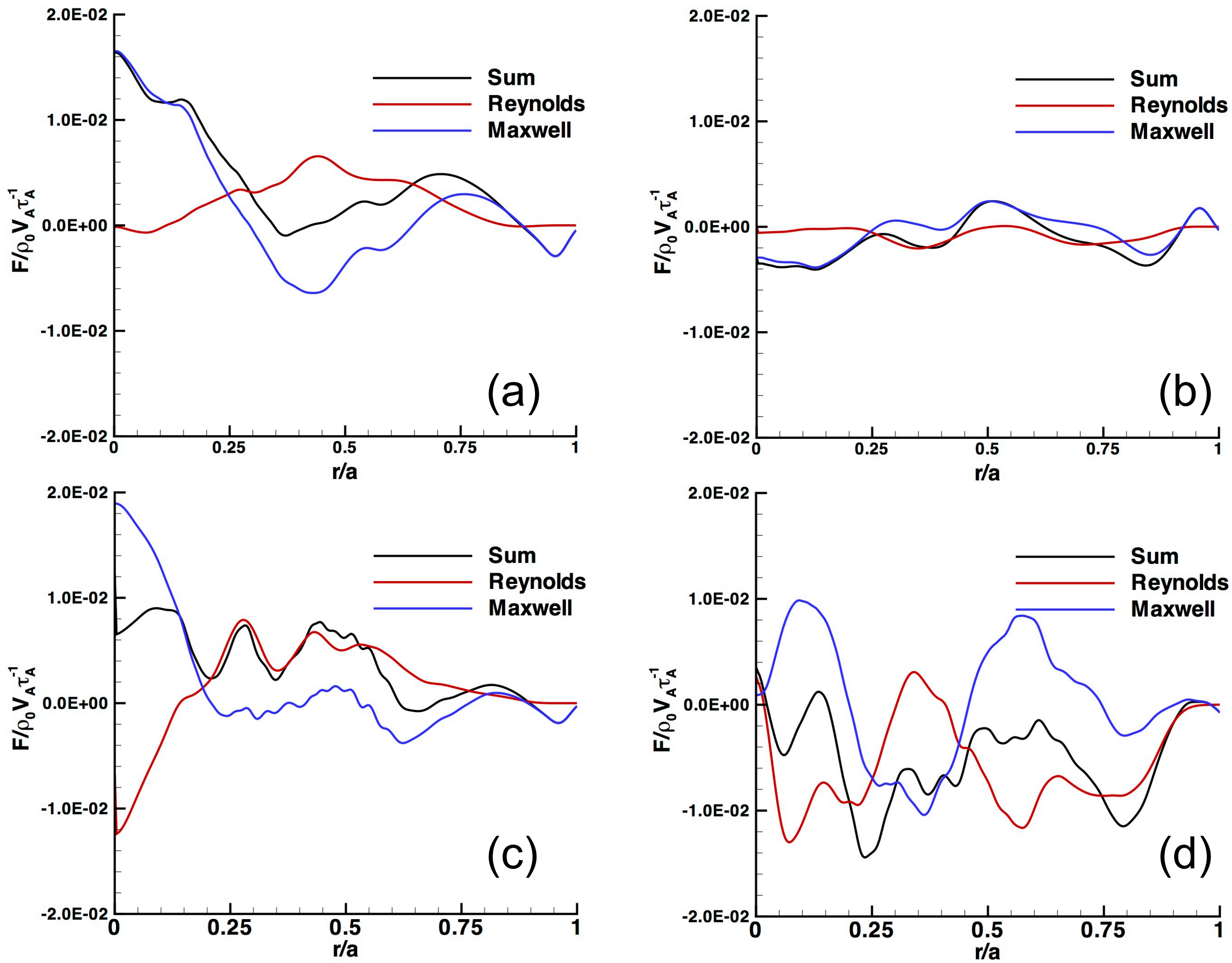
Flow does not affect dynamo from ideal-unstable modes.



Dynamo effects from ideal-mode results with $J_0 \bullet B_0 > 0$ (a) Hall with warm ions, (b) MHD with warm ions, (c) Hall with cold ions, (d) MHD with cold ions.

- Flow can change the sign of Hall & MHD dynamo from warm-ion tearing.

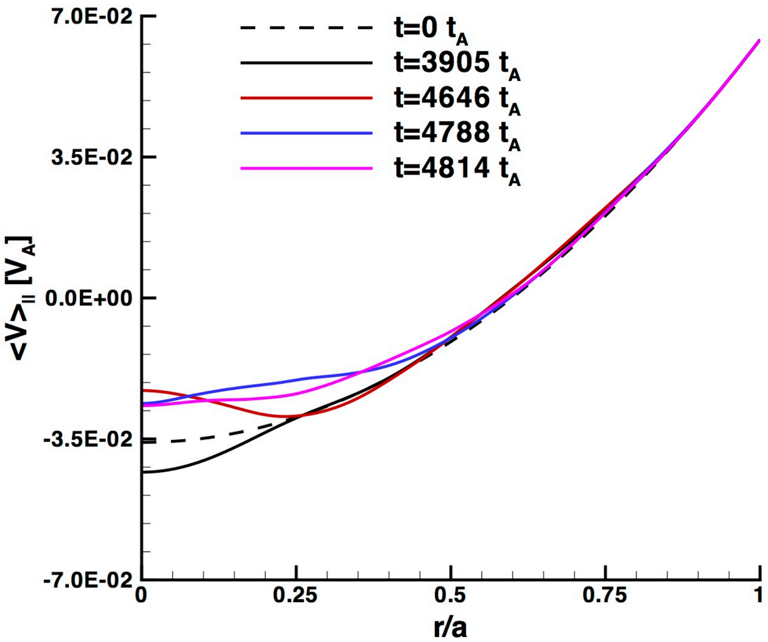
At reversal in the 3D cases, Maxwell-stress forces are stronger than Reynolds-stress forces for both $\mathbf{J_0}$ orientations.



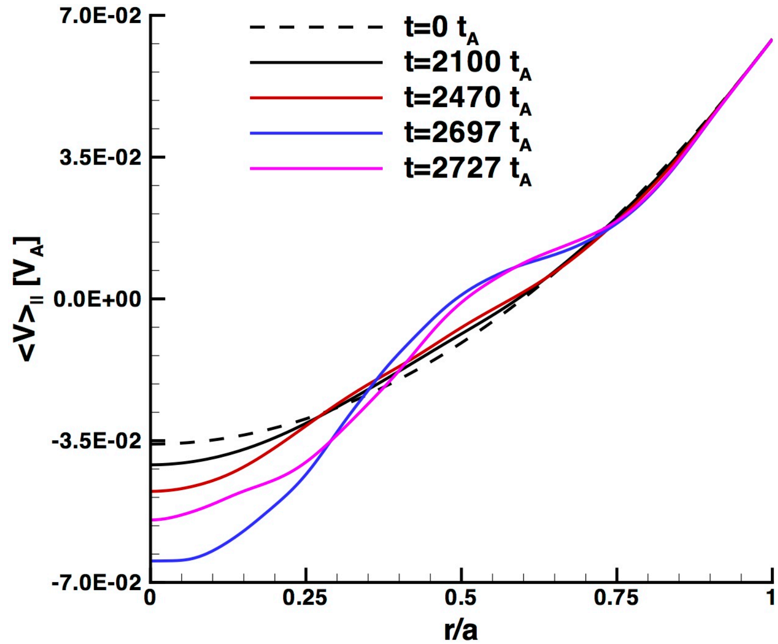
Fluctuation-induced parallel force-densities at initial reversal with (a) $J_0 \cdot B_0 > 0$ and (b) $J_0 \cdot B_0 < 0$ and at greatest reversal with (c) $J_0 \cdot B_0 > 0$ and (d) $J_0 \cdot B_0 < 0$.

Although Reynolds-stress contributions increase by the time of peak reversal, sequencing is important for the parallel flow profile evolution.

- For both orientations, parallel flow evolves in the direction of $\mathbf{J}_0$, consistent with Hall dynamo relaxation of the current profile.



Evolution of the parallel flow profile with positive current.



Evolution of the parallel flow profile with negative current.

- In MST, $\mathbf{J}_0 \bullet \mathbf{B}_0 < 0$, and this two-fluid computation evolves flow in the opposite direction of the observations in Ref. [Kuritsyn, PoP **16**, 55903.]

Conclusions

- First-order FLR modeling of warm ions produces ∇B and poloidal-curvature drift-tearing effects in pinch profiles.
  - At large-ρ_i, tearing decouples from ions, so ion kinetics may not be critical.
- Nonlinear island evolution in pinch profiles is influenced by ion FLR.
  - With cold ions, two-fluid saturation is equivalent to MHD.
  - When modeling experimentally relevant parameters with first-order FLR, warm-ion effects reduce island widths.
- Two-fluid multi-helicity simulations show non-MHD effects similar to those observed in MST: Hall dynamo and competition between Maxwell and Reynolds stresses.
- Background flows affect the profile evolution.
  - Both the Hall and MHD dynamo profiles differ from results without background flow.
  - Background flow breaks the symmetry between conditions with parallel and anti-parallel current.

Future Work

- Investigate profiles with nonuniform background pressure.
  - Incorporates pressure-driven effects.
  - Changes the ρ_s-profile to decrease in *r*.
- Include parallel viscosity in the model.
  - Fluctuations in $\hat{\mathbf{b}}$ and $\nabla_{||}\mathbf{V}$ can affect momentum transport.
- Investigate possible effects from the toroidal-field circuit.
- Apply the same modeling in toroidal geometry.