Energetic Particle Long Range Frequency Sweeping and Quasilinear Relaxation

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Abstract:

The relaxation of energetic particles with MHD modes is an area of need of investigation to enable credible planning for future self-sustained burning plasmas. Here we discuss two aspects to the relaxation: 1) the results of a simplified quasi-linear theory for energetic particle relaxation, which agree well with data in D-IIID; 2) the effect of frequently observed long-range frequency sweeping events attributed to the formation of clump and hole phase space structures.

1. Introduction

Here we discuss the following topics that are relevant to the issues of alpha particle transport in burning plasmas: Sec.(2). progress in the development quasi-linear (QL) techniques; Sec.(3). new approach to accurately describe the evolution of phase space structures and a mechanism for single linear wave excitation of phase space structures to cause global relaxation; Sec.(4). description of phase space structures that form in the TAE gap and may chirp into the continuum; Sec.(5). summary and comments of work presented here.

2. Quasilinear theory

The basic element for the improvement of a previously used QL model [1] is the employment of the linear stability NOVAK code [2] to calculate growth rates. Originally, analytic growth and damping rates for TAE were used to determine the critical energetic particle (EP) beta gradient profile that may lead to EP losses. We assume that the critical beta gradient is given by, $\partial\beta_{EP}(r)/\partial r|_{cr} = (\gamma_d/\gamma_{Lanlt})\,\partial\beta_{EP}(r)/\partial r$, where $\beta_{EP}(r)$ is the hot particle beta value at position r in the absence of QL relaxation, γ_{Lanlt} is the analytic linear growth rate in absence of dissipation and γ_d is the damping rate in absence of destabilizing sources. The linear growth rate has the form $\gamma_{Lanlt} = \gamma_L^*\partial\beta(r)/\partial r$ with γ_L^* independent of the EP profile. Hence in the region of the unstable mode there is QL relaxation of the beta gradient to $-\partial\beta\left(r\right)/\partial r \leq -\partial\beta_{EP}\left(r\right)/\partial r|_{cr}$. If $-\partial\beta\left(r\right)/\partial r$ predicted by the TRANSP code (which does not account for instability induced transport) is larger than this critical value $-\partial\beta_{EP}\left(r\right)/\partial r|_{cr}$, the profile is relaxed to $-\partial\beta_{EP}\left(r\right)/\partial r|_{cr}$, enabling a QL prediction of the EP beta gradient profile and EP losses. This relaxation allows for predictions of the neutron production deficit and other quantities.

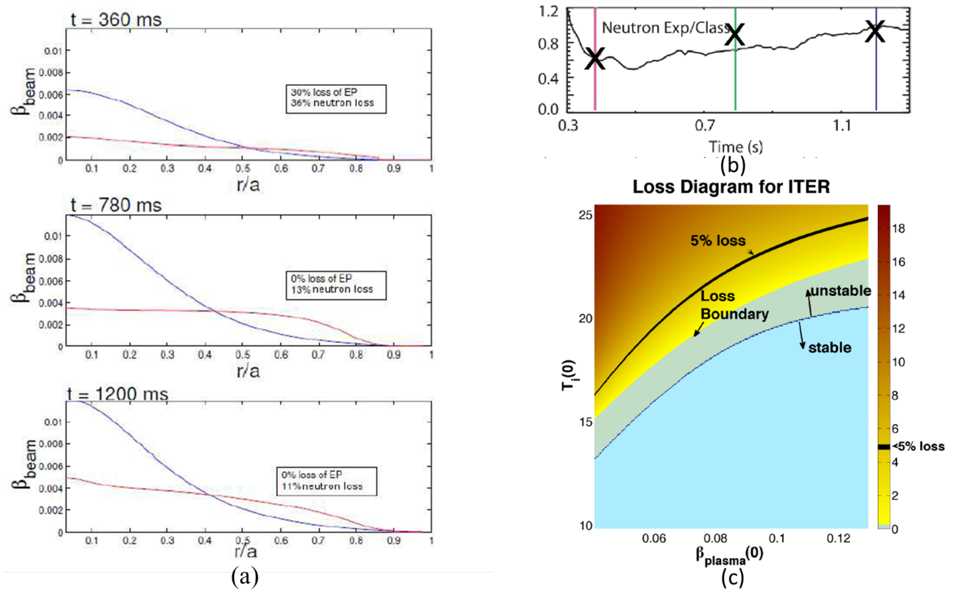


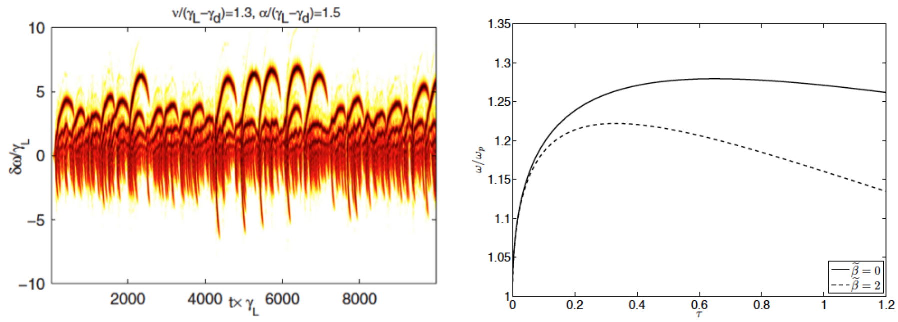
FIG. 1: a) Beam beta profile relaxation predicted by $1.5D$ *QL mode (red) and by the TRANSP code (blue curve) at:* $360ms$*(top),* $780ms$*(middle),* $1200ms$*(bottom); b)* $1.5D$ *QL prediction of relative neutron rate (X-points) compared with experimental rate (solid curve); c) ITER PopCon diagram indicating Alfvénic stability and instability regions and percentage loss of alpha particles.*

The $1.5D$ model improves the accuracy of the expressions for the growth and damping rates and is used together with the analytic estimate. Growth rates are calculated at two separated radial points, r_i, from both analytic and NOVA calculations. Then local analytic growth rates are altered by multiplying the analytically predicted growth rate by, $\gamma_{LNova}\left(r_i\right)/\gamma_{Lanlt}\left(r_i\right)$, at the two points, r_i, and then interpolating between the two points to produce localized growth rates in the domain $r_1 < r < r_2$. Only the pressure radial profile is relaxed. We account for velocity space relaxation by using rules based on the work of Kolesnichenko [2, 3]. With these changes the code captures the effect of the unstable Alfvénic modes on EP confinement. The $1.5D$ QL model is applied to a DIII-D experiments where tangential NBI excites Alfvénic-like modes [4]. A reversed magnetic shear plasma forms that produces EP induced Alfvénic excitations peaked around the shear reversal surface. Figure 1(a) shows the shape of the relaxed EP profile predicted by the $1.5D$ QL code (red curves) as well the shape and relative magnitude of the EP profile predicted by TRANSP (blue curves). The relaxation was examined at three specific times ($360ms$, $780ms$ and $1200ms$). There is significant loss of beam particles at $360ms$.(30%)

but at later times there is only redistribution of EPs without losses, causing ten to fifteen percent loss of neutron production. These results are compared with the solid curve in the fig.1(a), which is the measured neutron flux. The three X-points in figure 1(b) depict the QL predicted neutron flux. The continuous solid curve is the experimental neutron flux. It shows that the neutron flux reductions in the experimental data are comparable to QL predictions. Both the experiment and QL code predict an improvement of the neutron production at a later time of the instability. Time consuming numerical analysis can be avoided with use of just analytic expressions for the instability threshold. As a fusion exercise, we considered a nominal normal shear ITER plasma. We choose : identical parabolic temperature profiles for all the plasma species; central electron temperature $T_{e0} = 20keV$; a flat density profile for a 50/50 deuterium and tritium plasma. DT fusion rates are used to evaluate the fusion alpha source profile, which in turn leads to the AE instabilities and subsequent redistribution. The predictions shown in figure 1(c) is a PopCon diagram for the ITER plasma operational space. A more sophisticated modeling effort is attempting to predict relaxation of the entire phase space with a $2.5D$ QL formulation, that can treat cases where there is no mode overlap, partial overlap and full mode overlap [5].

3. Theory for long range frequency sweeping

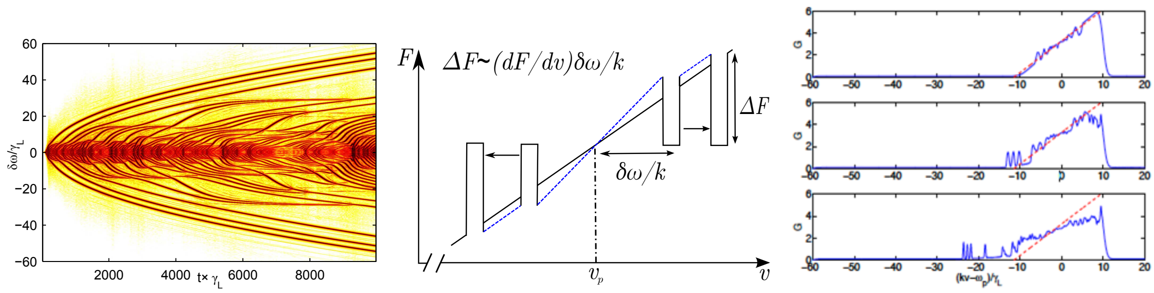
Commonly observed frequency sweeping events [6, 7] in plasmas with EP reflect the tendency of near-threshold kinetic instabilities to give rise to spontaneous formation of coherent phase space structures (holes and clumps) [8] in the energetic particle distribution function that produce frequency sweeps, where parameters change slowly compared to the trapped particle's bounce time. Thus a bounce-averaged description of the particle and wave evolution has been developed. The EPs trapped in the waves can be characterized by their adiabatic invariants that change due to collisions. The structures release of free energy balances the dissipative effects in the bulk plasma, enabling holes and clumps to persist for lifetimes much longer than the damping time of a linear wave in a dissipative plasma. To explain the large frequency changes often observed in experiment e.g. [6, 7], a non-perturbative theoretical formalisms has been developed in [9, 10]. The initial formation of coherent structures occurs spontaneously as a result of instability at the plasma eigenmode frequency. Through dissipation, energy is transferred from the wave to the background plasma without the phase space structure losing its coherency, while there is a significant frequency shift from the initial frequency. To evaluate the nonlinear response one uses that the only essential change in the distribution is within the trapped particle phase space area.The perturbed distribution function $\delta f(J;t)$ satisfies a bounce-averaged kinetic equation [10] $\frac{\partial \delta f}{\partial t} = -\beta\delta f - (\frac{\lambda}{2\pi}\alpha^2 + \frac{d^2 s}{dt^2})\frac{dF(V=\dot{s})}{dV} + \frac{m\lambda^2\nu^3}{4\pi^2}\frac{\partial}{\partial J}[J\frac{dJ}{d\epsilon}\frac{\partial \delta f}{\partial J}]$ where $J(\epsilon;t) = \frac{m}{\sqrt{2\pi}}\oint\sqrt{\epsilon - U(z;t)}dz$ is the adiabatic invariant. Particle collisions are modeled as a combination of velocity space diffusion, drag and annihilation with effective collision rates ν, α and β, respectively. This equation together with: the Poisson equation; linear fluid equation of a cold plasma; the power balance condition $m\dot{s}(\lambda\alpha^2/(2\pi) + d^2s/dt^2)\int_0^\lambda dz \int dV \delta f = (\gamma_d \omega_p^2 \lambda/2\pi e^2 \dot{s}^2) < U^2 >_\lambda$ describe the evolution of trapped particle distribution function and the wave phase velocity $\dot{s}$. This adiabatic model enables an efficient and accurate calculation of the coherent phase space structures on time scales larger than the trapped bounce period. Frequency sweeping on the order of the mode frequency can arise while the spatial field structure changes considerably. In addition, a new feature is revealed, non-monotonic (hooked) frequency sweeping. Hooked frequency sweeping was previously observed for holes during frequency sweeping using a fixed linear equilibrium slope, as a result of the interplay between drag and velocity space diffusion [11] shown in fig.2(a). During long-range frequency sweeping, variation in the slope of the equilibrium distribution function can produce hooks in the presence of drag alone and a distribution slope that varies with position as shown in fig.2(b).



(a) (b)

FIG. 2: Formation of 'hook' chirping structures due to (a) interplay between drag and diffusion, as observed in dynamical simulation; (b) 'hook' formed in adiabatic calculation with drag and velocity dependent slope (b).

A single low-amplitude linear mode with fixed frequency cannot cause global transport, because it affects only a small fraction of the EP population. According to conventional QL theory multiple modes with overlapping wave-particle resonances are required to produce global diffusion. However, it has been observed that a sequence of chirping events supported by an EP source can change the fast particle distribution globally [12]. More recently, it has been found [11], and shown in fig.3(a), that chirping events can persist even without a particle source to rejuvenate the unstable distribution at resonance. The underlying reason for this repetitive chirping is shown schematically in fig.3(b), which indicates that as chirping structure move in phase space, the untrapped particles jump across the separatrix over the resonance interval and leave a slightly steeper wake in the distribution function behind the structure. The consequence of this is that the particle distribution function remains unstable in the resonance region of the linear wave after a chirping structure is created and the transient response quiets down. Thus the system is ready to excite another unstable wave, which then generates another chirping structure. Hence, to the extent that the unperturbed distribution is of constant slope, there will be a continual generation of chirping signals [13]. The largest range possible is where the wave phase velocity leaves the velocity interval occupied by fast particles. In this case the flattening in the distribution that is achieved may be more than the relaxation predicted by the QL theory for a system near marginal stability. Thus the continuously produced non-perturbative holes and clumps provide a new channel for particle relaxation even when there are a limited number of linear modes excited. Figure 3(c) shows relaxation to an extended plateau in a simulation arising from hole formation when only a single unstable linear mode is present. It is pertinent to consider how this convective transport manifests itself in experiments.



(a) (b) (c)

FIG. 3: a) Continual chirping without particle source; b) cartoon explaining continual clump and hole formation; c) particle distribution functions during repetitive hole/clump production leading to plateau formation above the resonant velocity.

4. Long range chirping in a TAE model

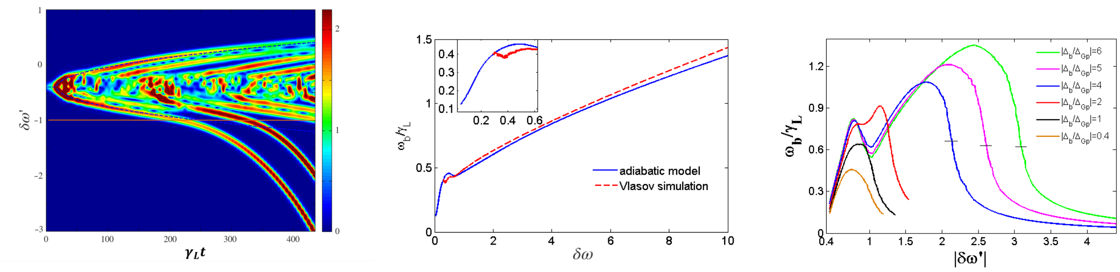
To describe a frequency chirping model that explicitly involves the TAE excitation, we generalize the theories, originally developed by Rosenbluth, et.al.[14], for the evolution of a TAE wave in a large aspect ratio low beta plasma, and the map model theory of ref. [15]. In the appendix given at the website (http://www.ph.utexas.edu/˜wange/appendix.pdf), we exhibit the Lagrangian governing the particle wave interaction and the Hamiltonian used in the needed set of Vlasov equations. Below we will refer to a gap region near the spatial point r_m, where $q(r_m) = (m + 1/2)/n$, $\omega_{TAE} \equiv k_{\|m}(r_m)V_A = -k_{\|m+1}(r_m)V_A = V_A/(2q(r_m)R)$, with V_A the Alfvén velocity, $k_{\|m}(r_m)$ the parallel wavelength at position r_m and R the major radius. The frequency gap width is $\Delta\omega_{TAE} \approx r_m\omega_{TAE}/R$. The gap's spatial width is $\Delta_{Gp} \approx r_m^2/(Rms_m)$ with s_m the local magnetic shear. The drift orbit width of a particle is Δ_b.

The principal difficulty in solving the entire set of Euler-Lagrange equations is that the structure of the spatial interaction is a relatively complicated function of the normalized momentum Ω and normalized frequency $\delta\omega'$ [$\delta\omega' \propto (\omega - \omega_{TAE})$, the O-point for the momentum is very close to $\Omega = \delta\omega'$]. As a first approach to this problem, we considered a model where the wave amplitude (whose magnitude taken as ω_b^2, with ω_b the particle trapping frequency at the O-point of the wave when the chirping rate is small), is independent of momentum and frequency. In this case, the treatment of the wave-particle interaction is identical to the bump-on-tail problem but now we include the linear wave evolution dynamics appropriate to TAE excitations. The TAE gap exists in the region $-1 \leq \delta\omega' \leq 1$, and there is an upper continuum region ($\delta\omega' > 1$) and lower continuum region ($\delta\omega' < -1$). In this case, chirping phase space structures first form within the TAE gap near the linear frequency. The results are shown in fig.4(a). The down chirping frequency signals, due to clumps smoothly enter the continuum, while the up-chirping frequency component, due to holes, eventually stagnate near the gap.

The simulation results are compared with the predictions of adiabatic theory [16]. The Hamiltonian used in this calculation accounts for the chirping parameter, $\alpha_c = (d\omega/dt)/\omega_b^2$, which varies within $|\alpha_c| < 1$ and need not be small. The constancy of the adiabatic invariant J is justified if the rate of change of the parameters α_c and ω_b is much smaller than the characteristic bounce frequency of trapped particles in a chirping structure. Then $f(J)$ is constant within the phase space structure, except for a small phase space region near the separatrix. We have numerically solved the adiabatic equations for ω_b and α_c as a function of $\delta\omega$. The input values for γ_L, (the growth rate when there is no background dissipation) and γ_d (the damping rate without an instability drive), are: $\gamma_L = 0.1, \gamma_d = 0.08$ for all cases presented here. As shown in fig.4(b), the adiabatic theory quantitatively replicates the dynamical simulation results for ω_b as of function of the chirping frequency for the downward chirping frequency of a clump. In the appendix on the web we show that the chirping parameter, α_c, is similarly replicated. A comparison with an upward chirp of a hole (see appendix), which does not penetrate the continuum, is also good in the region where comparisons can be made. For a damping model whose source of dissipation comes from the vicinity of the gap, the adiabatic theory predicts that the phase space hole chirps towards the continuum and the trapping frequency decreases but the chirping parameter increases towards unity. Analysis shows that when as $\alpha_c \to 1$ the adiabatic theory breaks down. The adiabatic theory accurately replicates the simulation until $\delta\omega' = 0.6$, whereupon the simulation structure suddenly disappears. The disappearance occurs when the adiabaticity criterion of the adiabatic calculation has already started to increase. Thus the adiabatic theory for the simplified simulation generally gives accurate predictions when the adiabaticity criterion is being satisfied.

The treatment of the problem using the systematically derived Lagrangian is more complicated. There are two new aspects to the difficulty. One is that the amplitude of the interaction term in the Hamiltonian depends on momentum (which is equivalent to position) and secondly the structure of the amplitude depends on frequency. Here we report on the results of the appropriate adiabatic theory we have derived for this problem. The solutions for a downward chirping clump are particularly sensitive to the parameter Δ_b/Δ_{Gp}. If $|\Delta_b/\Delta_{Gp}| < 1$, the mode amplitude vanishes as the continuum is approached, as indicated in Figure 4(c). However this figure does show pronounced chirping into the continuum when $|\Delta_b/\Delta_{Gp}| > 1$. But in contrast to the simplified model discussed in the previous paragraph, the frequency band in which there is a downward chirp is limited. Indeed, we can show that when $|\delta\omega'| < [(\Delta_b/\Delta_{Gp})^2 + 1]/|2\Delta_b/\Delta_{Gp}|$, a particle, due to its finite radial width, intersects two different spatial points of the continuum, while if $|\delta\omega'| > [(\Delta_b/\Delta_{Gp})^2 + 1]/|2\Delta_b/\Delta_{Gp}|$ the trapped particle intersects only one resonant point. At the point of equality, fig. 4(c) shows (see tic marks on the curve) that mode amplitudes are in the middle of a transition where there is a rapid fall off of the mode amplitude which effectively ends the chirping range. During this rapid change of mode amplitude, the requirement for justifying the adiabatic approximation is not clearly satisfied. Hence, in this region a dynamical simulation study is needed for a quantitative description. We also find, that for a down chirping clump near the lower continuum, when $|\delta\omega'| \ll [(\Delta_b/\Delta_{Gp})^2+1]/|2\Delta_b/\Delta_{Gp}|$, the form of the Hamiltonian is quite similar to the previously presented simple model (that produces unlimited chirping), if the particle orbit widths intersect both continuum points when the frequency is in the continuum. As in the previous model, an up-chirping hole does not penetrate into the upper continuum.

In experimental data, e.g. ref.[6], indicates stronger long range downward frequency chirping than upward chirping which is the case in our simulations. Our theory indicates that the chirping range is determined by the ratio of orbit width to mode width. This is a new mechanism for the limit of downward chirping that may apply to experiment. However experiments have other mechanisms that limit chirping that have not accounted for in our theory. These are: diffusion due to stochastic processes and the presence of a lower frequency gap due to the geodesic acoustic mode. Note that experimental data comes from small and moderate aspect ratio tokamaks, while the theory is for large aspect ratio tokamaks. Hence, quantitative comparisons cannot be made, but this theory can serve as a qualitative guide to understanding observed phenomena.



(a) (b) (c)

FIG. 4: a) Dynamical simulation of frequency chirping spectrum of TAE wave; b) adiabatic prediction of evolution the trapping parameter ω_b for a down-chirping clump as a function of frequency shift $\delta\omega \propto |\omega_L - \omega|$ for simplified theory; c) induced trapping frequency, ω_b of chirping structures for various ratios of orbit width to spatial width of gap $|\Delta_b/\Delta_{Gp}|$, as function of $\delta\omega'$, the normalized frequency shift from the linear frequency. Tic marks denote where trapped particle transition from two to one crossing of the continuum.

5. Summary and comments

The work presented shows several approaches now being attempted to understand the implications of the wave/alpha particle nonlinear interactions in a fusion plasma. A very rough 1.5D alpha particle model correlates with an interesting feature of a DIII-D experiment (shot #122117), the reduction of the expected neutron rate. The PopCon diagram displayed shows that it may be possible to operate in an unstable Alfvénic parameter region with alpha particles losses less than five percent, when the background plasma density changes by as much as 35%, and the ion temperature by 20%. This indicates that under transient conditions, particularly at start-up, running in a region unstable to Alfvénic waves may be acceptable. A deep question is, which theory is the most suitable description of alpha particle turbulence? QL theory might apply if collisional and other stochastic processes cause enough diffusion to prevent the formation of phase space structures [17] which would allow a steady spectral signal to be established. In contrast with sufficiently weak collisions, the generation of phase space structures are possible. The generation of chirping structures by a single linear wave can lead to continuous repetitive chirps that can cause global relaxation. This actually can have the favorable feature of being a mechanism for establishing energetic particle channeling that can transfer energy directly to the plasma without loss of energetic particles. A major accomplishment has been the formulation of a consistent method to compute phase space structure evolution using bounce averaged kinetic theory together with the wave equation for the system. A generalization of this procedure can in principle be accomplished for a description of phase space structures that might arise from Alfvénic excitations in a tokamak.

A model has been presented where TAE waves are excited in the TAE gap. These waves cause the formation of up-chirping hole structures (holes move toward the inside of a tokamak) and down-chirping clump structures (clumps move towards the outside of a tokamak). The clumps are able to penetrate into the lower continuum, but the holes remain confined to the gap. The adiabatic theory is found to quantitatively agree with the simulation results. A rigorous model, based on interaction forms whose mode amplitudes depend on momentum and frequency, has been formulated and the adiabatic theory of this formalism has been solved. It shows: upward chirping holes are still confined to remain in the TAE gap; while downward-chirping clumps can penetrate deeply into the lower continuum, but only if the orbit width of resonant particle is sufficiently large. The range of validity of the systematic developed TAE theory is very restrictive (e.g. large aspect ratio). Hence, the present results can serve as a guide to our understanding of TAE chirping into the continuum, but new theoretical extensions are needed to be able to compute, with reduced dynamics, chirping phenomena that can quantitatively be compared with experiments.

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